$\sigma_t^2$ = dimensionless variance, sq. sec.

**Subscripts**

$B$ = standard 90 deg. elbow
$P$ = straight pipe

### LITERATURE CITED

1. Hawthorn, R. D., *AIChE J.*, **6**, 443 (1960).
2. Hull, D. E., and J. W. Kent, *Ind. Eng. Chem.*, **44**, 2745 (1952).
3. Levenspiel, O., *Petrol. Refiner*, **37**, 191 (1958).
4. ———, *Ind. Eng. Chem.*, **50**, 343 (Mar., 1958).
5. Sjenitzer, F., *Pipeline Eng.*, **30**, D31 (Dec., 1958).
6. Taylor, G. I., *Proc. Roy. Soc. (London), Ser. A*, **223**, 446 (1954).
7. Tichacek, L. J., C. H. Barkelew, and T. Baron, *AIChE J.*, **3**, 439 (1957).
8. Turner, J. C. R., *British Chem. Engr.*, **9**, 376 (1964).
9. Yablonski, V. S., A. Sh. Asaturyan, and I. Kh. Khizgilov, *Intern. Chem. Engr.*, **2**, 3 (Jan., 1962).
10. Aris, R., et al., *Chem. Eng. Sci.*, **9**, 266 (1959).
11. Bischoff, K. B., et al., *ibid.*, **12**, 69 (1960).
12. Levenspiel, O., and W. K. Smith, *ibid.*, **6**, 227 (1957).
13. Bischoff, K. B., and O. Levenspiel, *ibid.*, **17**, 245 (1962).
14. *Ibid.*, **17**, 257 (1962).
15. Bischoff, K. B., *AIChE J.*, **10**, 584 (1964).
16. Halwagi, M. M., and A. Gomezplata, *ibid.*, **13**, 503 (1967).
17. Bischoff, K. B., *Can. J. Chem. Eng.*, **41**, 129 (June, 1963).
18. Cassell, R. E., MS thesis, Univ. Tennessee, Knoxville (1966).
19. Wilks, S. S., "Elementary Statistical Anlysis," pp. 116-117, Princeton Univ. Press, Princeton, N.J. (1948).
20. Koutsky, J. A. and R. J. Adler, *Can J. Chem. Eng.*, **42**, 239 (Dec. 1964).
21. Carter, D. and W. G. Bir, *Chem. Eng. Progr.*, **58**, 40 (March, 1962).

*Manuscript received May 29, 1967; revision received January 25, 1968; paper accepted January 26, 1968.*

---

# Evaluation of Drying Schedules

**RALPH E. PECK and JAE Y. KAUH**

**Illinois Institute of Technology, Chicago, Illinois**

**A generalized drying program that can simulate any drying conditions has been solved on a computer. The program includes the variation of humidity, temperature, and wind velocity over drying material. These three variables can be treated as functions of time. The correlations require knowledge of several basic properties of the material to be dried and were solved for the case of one directional heat and mass flow.**

**The computer calculated results are checked with experimental data. The effective diffusivity and the solid surface resistance to mass transfer were evaluated by data obtained in three experiments.**

**Solutions for constant drying conditions are presented in graphical form. Variations of such variables as humidity in a drier can be estimated from these curves but the computer program is more reliable.**

The process of the drying of solids consists of both the transfer of moisture within the solid and the evaporation of moisture from the surface of the solid into the surrounding medium.

### MOISTURE TRANSFER IN THE SOLID PHASE

The transfer of moisture within the drying body is caused by various forces. Such forces may be external pressure, gravity, capillary tension, and molecular diffusion forces. The importance of these various forces is dependent upon the material and drying conditions.

Experimental data for the moisture distribution in various kinds of material have been taken and analyzed by many investigators (*1* to *5*).

In low temperature drying it can be assumed that most of the moisture diffuses through the solid phase as liquid. The simplest form of the drying differential equation in the falling rate period is

$$\frac{\partial \not{C}}{\partial t} = \alpha \frac{\partial^2 \not{C}}{\partial x^2} \qquad (1)$$

where $\alpha$ is the effective diffusivity of free moisture within the drying solid, $C$ is the free moisture concentration, weight of free moisture per unit weight of bone dry solid.

Sherwood (*6* to *9*), Newman (*10, 11*), Ceaglske and Hougen (*12*), and others have recommended this equation for predicting the movement of water in a solid or have presented solutions to the equation.

The diffusion equation can be used in all cases where the major resistance is in the gas phase, and in cases where the equation holds in the solid. This equation seems to apply quite well for wood.

### MOISTURE TRANSFER ON THE DRYING SURFACE

$\not{C} = \not{C}_o$ is taken as an initial condition. The boundary conditions at the surface $x = L$ are taken as

$$-\alpha \rho_o \frac{\partial \not{C}}{\partial x} = k_g \gamma (P_s - P_g) \qquad (2)$$

and

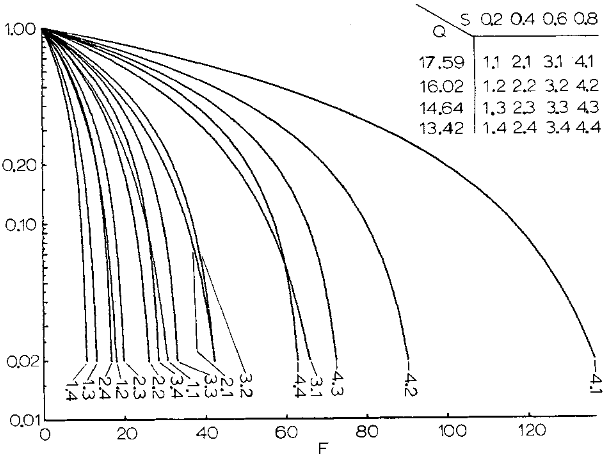

Fig. 1. Dimensionless drying curves at Z = 1.0.

---

Jae Y. Kauh is with Union Carbide Co., South Charleston, West Virginia.

$$-\alpha\,\rho_o \frac{\partial \text{¢}}{\partial x} = \frac{h}{\lambda}\,(T_g - T_s) \qquad (3)$$

$k_g\gamma$ is a variable parameter which accounts for the mass transfer resistance at the surface. In the falling rate period the solid occupies some of the area so that only part of the area is available for mass transfer. The evaporating water may fall slightly below the top of the surface which will increase the resistance to mass transfer.

If the molar volume of the water remains constant the area for evaporation would vary as the square of the equivalent wet length. The moisture concentration would vary as this length cubed. The ratio of the mass transfer area to the total area could be represented by

$$\left(\frac{\text{¢}_s}{\text{¢}_r}\right)^{2/3}$$

The assumption of constant molar volume of the water may not hold for some materials and the water surface may recede as drying progresses. For this work the mass transfer area and coefficient was represented by the equation:

$$\gamma = \left(\frac{\text{¢}_s}{\text{¢}_{cr}}\right)^{Y} \qquad (4)$$

The parameter $Y$ in the function $\gamma$ was determined empirically. The solution is not sensitive to the value of $Y$ and it was found that $Y = 0.6$ was a satisfactory value when computing drying times for balsa wood.

The vapor pressure on the surface of the moisture can be represented by

$$P_s = P_g{}^o \exp\left\{-\frac{\lambda}{R}\left(\frac{1}{T_s} - \frac{1}{T_g}\right)\right\} \qquad (5)$$

Where $\lambda$ is the latent heat, $R$ is the gas constant and $P_g{}^o$ is the vapor pressure of the evaporating liquid at the dry bulb temperature.

By substituting Equation (5) in the boundary condition (2) gives

$$-\alpha\,\rho_o \frac{\partial \text{¢}}{\partial x} = k_g\,\gamma\left[P_g{}^o \exp\left\{-\frac{\lambda}{R}\left(\frac{1}{T_s} - \frac{1}{T_g}\right)\right\} - P_g\right] \qquad (6)$$

Eliminating $T_s$ from Equation (3) and (6) gives

$$-\alpha\,\rho_o \frac{\partial \text{¢}}{\partial x} = k_g\,\gamma\left[P_g{}^o \exp\left\{\frac{\lambda}{R\,T_g}\left[\frac{\dfrac{\lambda}{h}\,\alpha\,\rho_o \dfrac{\partial \text{¢}}{\partial x}}{T_g + \dfrac{\lambda}{h}\,\alpha\,\rho_o \dfrac{\partial \text{¢}}{\partial x}}\right]\right\} - P_g\right] \qquad (7)$$

Equation (1) and (7) can be written in dimensionless form as follows:

$$\frac{\partial C}{\partial F} = \frac{\partial^2 C}{\partial X^2} \qquad (8)$$

and

$$\text{at } F = 0 \qquad C = I \qquad (9)$$

and

$$\text{at } X = \pm 1 - \frac{\partial C}{\partial X} = P\,\gamma\left[\exp\left[\frac{Q\,Z \dfrac{\partial C}{\partial x}}{1 + Z \dfrac{\partial C}{\partial X}}\right] - S\right] \qquad (10)$$

where

$$Q = \frac{\lambda}{R\,T_g} \qquad \text{(dimensionless temperature)} \qquad (11)$$

$$S = \frac{P_g}{P_g{}^o} \qquad \text{(humidity)} \qquad (12)$$

$$X = \frac{x}{L} \qquad \text{(dimensionless distance)} \qquad (13)$$

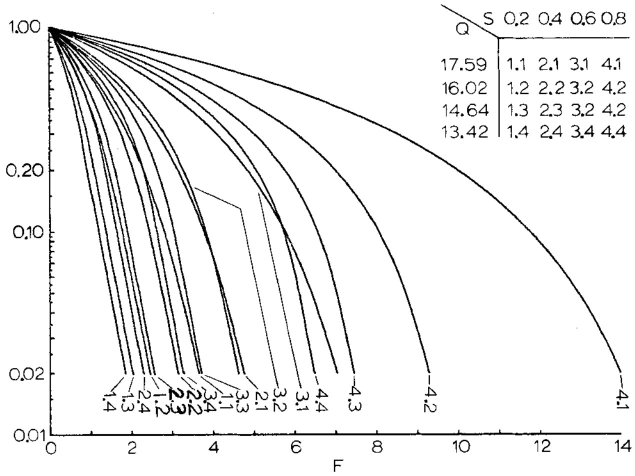

Fig. 3. Dimensionless drying curves at $Z = 0.1$.

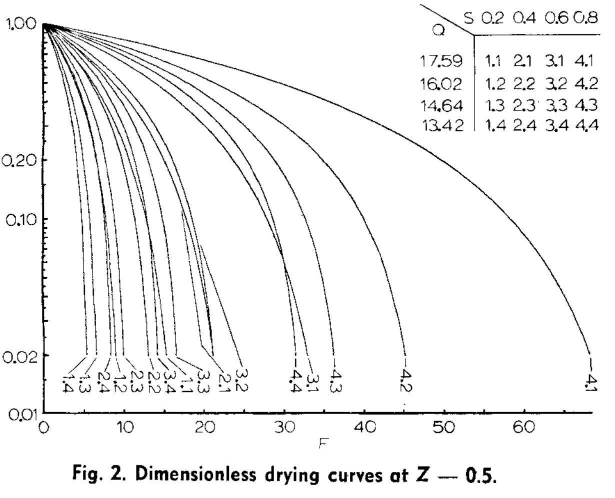

Fig. 2. Dimensionless drying curves at $Z$ — 0.5.

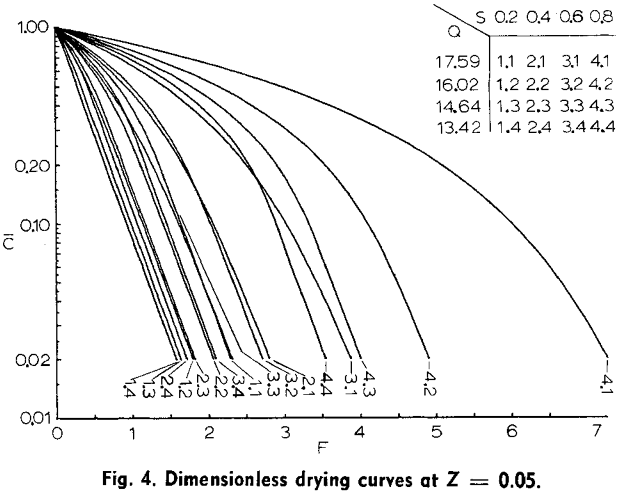

Fig. 4. Dimensionless drying curves at $Z = 0.05$.

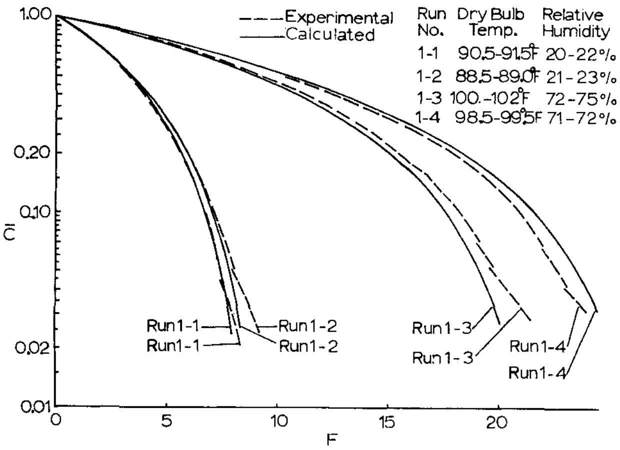

**Fig. 5. Experimental and calculated drying curves of a Balsa wood with a thickness of ⅛ in.**

$$F = t\left(\frac{\alpha}{L^2}\right) \quad \text{(dimensionless time)} \qquad (14)$$

$$Z = \frac{\rho_o\,\alpha\,\text{¢}_o\,\lambda}{h\,T_g\,L} \quad \text{and} \qquad (15)$$

$$P = \frac{k_g\,P_g{}^o\,L}{\rho_o\,\alpha\,\text{¢}_o} = \frac{1}{Z}\left(\frac{k_g}{h}\right)\left(\frac{P_g{}^o\lambda}{T_g}\right) \qquad (16)$$

$P$ was used in the computer program but is not needed in the general solution as $P$ is only a function of the psychrometric ratio, $Z$ and $Q$.

The parameter $Y$ in the function $\gamma$ has to be determined empirically. The solution is not sensitive to the value of $Y$ and it was found that $\alpha = 10^{-4}$ sq.ft./hr. is a satisfactory value when computing drying times for balsa wood.

Accurate values of the psychrometric ratio were needed for a flat plate. The values computed by Kauh (*13*) were used in this computation.

The program for the digital computer was developed and presented by Kauh (*13*).

Generalized solutions to Equation (8) for the drying of wet solid with water are presented in Figures 1 to 4. These solutions are for constant drying conditions of air temperature, humidity, and air velocity with initially saturated material. These results are for air temperatures from 80 to 200°F. and relative humidities of 20, 40, 60, and 80%. Solutions with small variations in air temperature and humidity can be handled with the above graphs. The figures are limited to material where $Y$ is approximately 0.6.

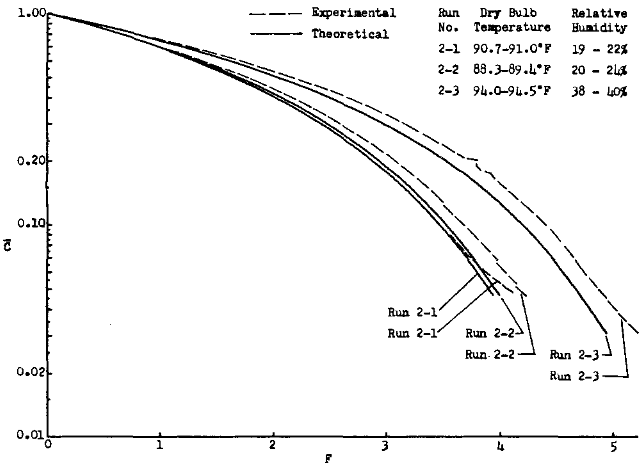

**Fig. 6. Experimental and theoretical drying curves of a balsa wood with a thickness of ¼″.**

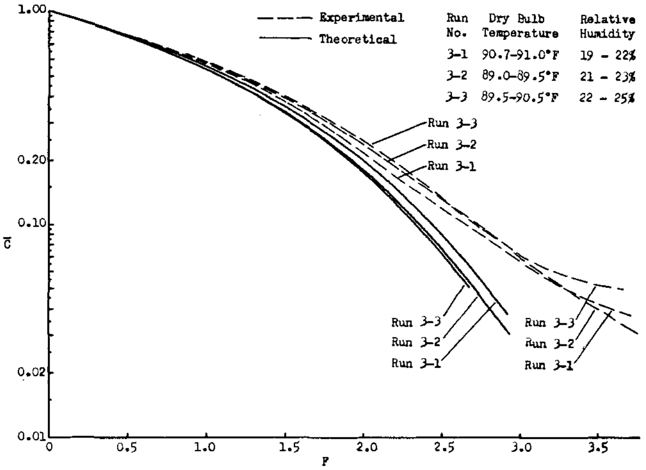

**Fig. 7. Experimental and theoretical drying curves of a balsa wood with a thickness of ⅜″.**

Use of the computer program is recommended when the independent parameters change rapidly with time.

The generalized correlations were compared with experimental results. Three experiments were used to evaluate $Y$ and $\alpha$ and the remainder were performed as a check on the calculations.

The data was obtained in a wind tunnel designed by Uno (*4*). Temperature, air velocity and humidity were controlled independently.

One-eighth, one-quarter, and three-eighth inch balsa wood slabs were dried at several temperatures, air velocities, and humidities. The results are shown in Figures 5, 6, and 7. The maximum deviation was 13% in the ⅜ in. slabs. This deviation could come from the variations in $\alpha$ with temperature and moisture concentration or from the correlation of $Y$.

## CONCLUSION

1. New boundary conditions were applied to the solution of the differential equation of drying. These conditions are more realistic than any of those proposed in the literature.
2. A drying model with five dimensionless parameters $P$, $Q$, $Z$, $S$, and $Y$ was programmed on a digital computer. Knowing certain properties of the material and its drying conditions, this program can be used to predict any drying schedule.
3. The developed digital computer program is capable of predicting drying schedules for cases where humidity, temperature and other independent variables are functions of time. This is the case with most industrial dryers.
4. Dimensionless plots of drying schedules at 20, 40, 60, and 80% relative humidity were prepared.
5. Drying experiments with balsa wood were performed and results were compared with schedules which were obtained from the computer calculated results. The results show that the calculated drying schedules agree with the experimental data with a maximum deviation of 13%.
6. The predicted value of $Y = 2/3$ checks the empirical value of 0.6 for balsa wood.
7. The only physical property of the balsa wood which was needed was $\alpha$ ($\alpha = xo^{-4}$ sq.ft./hr.) This figure accounts for the interval resistance for the flow of moisture in the solid.

## ACKNOWLEDGMENT

The authors are grateful to the Chicago Bridge and Iron Company for the financial support of this work.

## NOTATION

$C$ = dimensionless moisture concentration; $C/C_o$
$\overline{C}$ = an average value of $C$ over the thickness
$\text{₵}$ = free moisture concentration; weight of free moisture per unit weight of bone dry solid
$\text{₵}_{cr}$ = critical moisture concentration
$\text{₵}_s$ = free moisture concentration on the surface
$F$ = dimensionless time as defined by Equation (14)
$h$ = heat transfer coefficient
$k_g$ = convective mass transfer coefficient
$L$ = half thickness of a slab
$P$ = a dimensionless parameter as defined by Equation (16)
$P_g$ = vapor pressure of evaporating liquid in the gas stream
$P_g{}^o$ = vapor pressure of evaporating liquid at the dry bulb temperature
$P_s$ = vapor pressure of evaporating liquid at the surface temperature, $T_s$
$Q$ = dimensionless temperature as defined by Equation (11)
$R$ = gas constant
$S$ = relative humidity
$T_g$ = dry bulb temperature of gas stream
$T_s$ = surface temperature
$t$ = time
$X$ = dimensionless distance of $x$ as defined by Equation (13)
$x$ = distance from the center of a slab toward surface
$Y$ = an experimental constant as defined by Equation (4)
$Z$ = a dimensionless parameter as defined by Equation (15)
$\alpha$ = effective diffusivity of free moisture within the drying solid
$\gamma$ = correction factor for effective mass transfer area and coefficient as defined by Equations (2) and (4)
$\lambda$ = latent heat
$\rho_o$ = density of bone dry solid

## LITERATURE CITED

1. Marshall, McCauley, and Hougen, *Trans., AIChE*, **36**, 183 (1940).
2. Kamei, and Shiome, *J. Soc. Chem. Ind. (Japan)*, **40** (1937).
3. Bateman, Hohf and Stamm, *Ind. Eng. Chem.*, **31**, 1150 (1929).
4. Ceaglske, and Hougen, *Trans. AIChE*, **33**, 283 (1937).
5. Tuttle, E. E., *Franklin Inst.*, **200**, 609 (1925).
6. Sherwood, T. K., *Ind. Eng. Chem.*, **21**, 12 (1929).
7. *Ibid.*, 976 (1929).
8. Sherwood, T. K., *Trans. AIChE.*, **23**, 28 (1929).
9. *Ibid.*, **27**, 190 (1931).
10. Newman, A. B., *Trans. AIChE.*, **27**, 310 (1931).
11. *Ibid.*, 203-216, (1931).
12. Ceaglske and Hougen, *Trans. AIChE.*, **33**, 283-314 (1937).
13. Kauh, J. Y., Ph.D. dissertation, Illinois Institute of Technology, Chicago (1966).
14. Uno, S. E., Ph.D. thesis, Illinois Institute of Technology, Chicago (1958).

*Manuscript received April 31, 1967; revision received December 12, 1967; paper accepted January 8, 1968. Paper presented at AIChE Salt Lake City meeting.*

---

# Surface Rheological Properties of Foam Stabilizers in Nonaqueous Liquids

**R. J. MANNHEIMER**
**Southwest Research Institute, San Antonio, Texas**

**The rheological properties of foam stabilizers have been measured with an improved canal viscometer that provides absolute values of surface shear viscosity and yield strength. Studies conducted in room air confirmed previous reports of complex non-Newtonian film properties for oil soluble surfactants at the air-oil interface. Both shear and time dependent behavior were observed in which the apparent surface viscosity increased with decreasing rotational speed. Surface viscosities were low at all fresh interfaces but generally increased with the age of the surface over a period of several hours to several days. Experiments conducted in a controlled humidity environment have established that the phenomenon of aging in room air is due to adsorption of moisture from the gas by the surfactant film. Preliminary results indicate that the shear dependency of films in a high humidity environment can be approximated by a Bingham Plastic model which characterize their rheological behavior in terms of a Newtonian surface viscosity and yield value.**

In 1962, the U.S. Army Fuels and Lubricants Research Laboratory began a basic study of the various factors that affect the foaming of gear oils. This paper, which deals with the rheological properties of surfactant films that form at the air-oil interface, presents the results of a part of the program that have been found to be of primary importance to the foam stability of nonaqueous systems. During the course of this investigation, an improved viscometer was developed that enables one to calculate absolute values for the surface shear viscosity and yield strength of soluble films. The mathematical development of the equations, which relate these surface properties to experimental parameters, was the subject of an earlier paper by Burton and Mannheimer (*1*); however, this present paper is the first to report measured values of surface viscosity and rigidity that have been determined by this technique.

## THE ROLE OF SURFACE RHEOLOGY IN FOAMING

The concept of surface viscosity was first proposed by